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$$=\frac{5\sin\beta}{3a^2h^7}\int_0^h\int_r^h\int_{-y'}^{y'}\int_0^r(4a^2u^3+3a^2u^2v+3a^2uv^2+3h^2uy^2)(v-u)^2drdvdydu$$

$$=\frac{5\sin\beta}{36a^2h^7}\int_0^h\int_r^h\int_{-y'}^{y'}[3a^2r^2v^2-12a^2rv^3+18a^2v^4-12a^2r^3v+8a^2r^4$$

$$+3h^2y^2(6v^2-8rv+3r^2)]r^2drdvdy$$

$$=\frac{5a\sin\beta}{9h^8}\int_0^h\int_r^h(3r^4v^3-10r^3v^4+12r^2v^5-6r^5v^2+4r^6v)drdv$$

$$=\frac{5a\sin\beta}{36h^8}\int_0^h(3r^2h^4-8rh^5+8h^6-8r^3h^3+8r^4h^2-3r^6)r^2dr$$

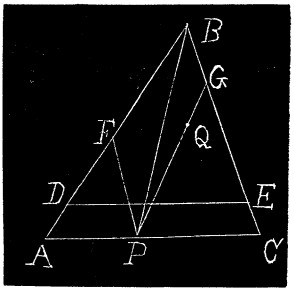
$$=\tfrac{13}{126}ah\sin\beta=\tfrac{13}{126}\text{ (area of triangle).}$$

118. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Find the mean distance between two points taken at random in an equilateral triangle.

I. Solution by L. C. WALKER, A. M., Graduate Student, Leland Stanford University, Cal.

Let ABC be any plane triangle whose sides opposite the angles A, B, C are respectively a, b, c. Let M be the required mean distance, and M_1 be the mean distance between the two points when one of the points is confined to one of the sides of the triangle, as AC. Let P be any point in the side AC, and let Q be any point in the surface of the triangle whose area$=\triangle$. Draw DE parallel to AC, and draw PF, PB, and PQG. Put $BD=u$, $AP=x$, $PQ=y$, $PF=z$, $PG=v$, $\angle BPC=\phi$, and $\angle FPC$ or $\angle GPC=\theta$. Then



$$v=\frac{c\sin A\sin(\phi+C)}{\sin\phi\sin(\theta+C)},\quad z=\frac{c\sin A\sin(\phi-A)}{\sin\phi\sin(\theta-A)},$$

$$x=\frac{c\sin(\phi-A)}{\sin\phi},\text{ and }dx=\frac{c\sin A}{\sin^2\phi}d\phi.$$

$$\text{Hence }M=\frac{\int_0^c M_1.\frac{u}{c}.\frac{u^2}{c^2}\triangle.DE.du}{\int_0^c\frac{u^2}{c^2}\triangle.DE.du}=\tfrac{4}{5}M_1$$

$$=\frac{4}{5b\triangle}\int_0^b\left[\int_0^\phi\int_0^v y^2dyd\theta+\int_\phi^\pi\int_0^z y^2dyd\theta\right]dx$$

$$=\frac{8c^3\sin^3A}{15b^2}\int_A^{\pi-C}\left[\int_0^{\phi}\frac{\sin^3(\phi+C)}{\sin^3\phi}\cdot\frac{d\theta}{\sin^3(\theta+C)}\right.$$

$$\left.+\int_{\phi}^{\pi}\frac{\sin^3(\phi-A)}{\sin^3\phi}\cdot\frac{d\theta}{\sin^3(\theta-A)}\right]\frac{d\phi}{\sin^2\phi}=\frac{4c^3\sin^3A}{15b^2}\int_A^{\pi-C}\left[\frac{\cos C\sin^3\,(\phi+C)}{\sin^2C\sin^3\phi}\right.$$

$$+\frac{\cos A\sin^3(\phi-A)}{\sin^2A\sin^3\phi}-\frac{\sin(\phi+C)\cos(\phi+C)}{\sin^3\phi}+\frac{\sin(\phi-A)\cos\,(\phi-A)}{\sin^3\phi}$$

$$+\frac{\sin^3(\phi+C)}{\sin^3\phi}\log\,\cot\tfrac{1}{2}C+\frac{\sin^3(\phi-A)}{\sin^3\phi}\log\cot\tfrac{1}{2}A\,-\frac{\sin^3(\phi+C)}{\sin^3\phi}\log\,\cot\tfrac{1}{2}(\phi+C)$$

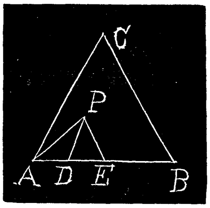
$$\left.+\frac{\sin^3(\phi-A)}{\sin^3\phi}\log\,\cot\,\tfrac{1}{2}(\phi-A)\right]\frac{d\phi}{\sin^2\phi}=\tfrac{1}{15}\left[\left(\frac{a^2}{b}+\frac{b^2}{a}\right)(\cos C+\sin^2C\log\,\cot\tfrac{1}{2}C)\right.$$

$$\left.+\left(\frac{a^2}{c}+\frac{c^2}{a}\right)(\cos B+\sin^2B\log\,\cot\tfrac{1}{2}B)+\left(\frac{b^2}{c}+\frac{c^2}{b}\right)(\cos A+\sin^2A\log\cot\tfrac{1}{2}A)\right].$$

Corollary. If $a=b=c$, then $M=\frac{a}{20}(4+3\log3)$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $\triangle$ be the required average, $\triangle_0$ the average distance from an angle to a point taken at random within the triangle. Let $AP=r$, $AB=a$, $\angle PAB=\theta$. The limits of r are 0 and $\frac{1}{2}a\sqrt{3}$ cosec $(\frac{2}{3}\pi-\theta)=x$; of θ, 0 and $\frac{1}{3}\pi$.



$$\triangle_0=\frac{4}{a^2\sqrt{3}}\int_0^{\frac{1}{3}\pi}\int_0^{x}r^2\,d\theta dr=\tfrac{1}{2}a\int_0^{\frac{1}{3}\pi}\text{cosec}^3(\tfrac{2}{3}\pi-\theta)d\theta$$

$$=a(\tfrac{1}{3}+\tfrac{1}{4}\log3).$$

From P draw PD, PE parallel to the sides and let the area of ABC=unity. The average distance from P to any point in PDE is $u(\frac{1}{3}+\frac{1}{4}\log3)$ where $u=DE$. Since either of the vertices of PDE can be taken,

$$\triangle=6\int_0^{a}\int_0^{a-x}u(\tfrac{1}{3}+\tfrac{1}{4}\log3)u^2\,dudv=\tfrac{3}{5}a(\tfrac{1}{3}+\tfrac{1}{4}\log3).$$

$$\therefore\ \triangle=\tfrac{3}{5}\triangle_0.$$

119. Proposed by F. L. SAWYER, Mitchell, Ontario, Canada.

Two players throw three dice, the object being to throw at one cast a tr and an ace. They continue throwing in succession until one of the players wi advantage has the first player?